

# Theory and characteristics of transition radiation emitted by low-energy storage-ring synchrotrons for use in X-ray lithography

D. Minkov,<sup>a\*</sup> H. Yamada,<sup>a,b,c</sup> N. Toyosugi,<sup>c</sup> T. Yamaguchi,<sup>b</sup> T. Kadono<sup>b</sup> and M. Morita<sup>c</sup>

<sup>a</sup>21st Century COE SLLS, 1-1-1 Nojihigashi, Kusatsu-shi, Shiga 525-8577, Japan, <sup>b</sup>Ritsumeikan University, 1-1-1 Nojihigashi, Kusatsu-shi, Shiga 525-8577, Japan, and <sup>c</sup>PPL Company, 1-1-1 Nojihigashi, Kusatsu-shi, Shiga 525-8577, Japan. E-mail: minkov@se.ritsumei.ac.jp

Existing theory is developed further for description of transition radiation (TR) emitted by low-energy storage-ring synchrotrons. It takes into account the fact that the dielectric constant of the TR target material is a complex function, introduces an expression for the number of passes of an injected electron through the target, and accounts more precisely for the absorption of TR. It is shown that the consideration of the complexity of the dielectric constant results in notable changes of the TR spectrum for emitted photons with energies close to the ionization energies of the target material. Since such TR is used mostly for performing X-ray lithography (XRL), the sensitivity of the photoresist used in XRL is formulated. Maximization of this resist sensitivity can be used for designing optimum targets for XRL. Study of the transmission of TR through a commonly used XRL mask, and its partial absorption in a common photoresist, illustrates that TR emission with  $E = [490, 1860]$  eV is most useful for performing such XRL, while  $E \cong 1$  keV is best. It is shown that, for a particular target material, a target consisting of only one foil emits the most TR energy. Optimization of an Al target, based on maximization of the resist sensitivity, indicates that a target containing one Al foil with a thickness of about 200 nm would be best for performing XRL by our low-energy storage-ring synchrotron MIRRORCLE-20SX.

## 1. Introduction

Transition radiation (TR) is emitted when a relativistic particle passes through the interface between two different materials (Ter-Mikaelyan, 1961; Garibyan, 1961). The main sources of TR are electron synchrotrons (SR), in which TR is emitted by passing relativistic electrons through appropriate material targets (TR targets).

Linear accelerators (LINACs) operate at very high electron energies  $E_{e1} \simeq$  GeV, and their relativistic electrons move along linear trajectories. Such LINACs have been mostly used for emission of TR (Piestrup *et al.*, 1985). The targets have consisted of several foils made of the same material and having the same foil thickness (Ebert *et al.*, 1985; Pianetta *et al.*, 1985). Thus there is a well developed theory of emission of TR by LINACs for targets consisting of several foils (Piestrup, Boyers, Pincus, Harris, Maruyama *et al.*, 1991).

The main disadvantages of using LINACs for TR emission is that their sizes are large, of the order of hundreds of metres,

they are expensive facilities, and the number of LINACs in the world is limited. Correspondingly, users cannot have their own LINACs. Another main disadvantage is that the electrons pass only once along their trajectory, limiting the beam current and therefore the emitted TR power since the TR power is proportional to the beam current.

More recently, low-energy storage-ring SRs have been used for TR emission. They operate at lower electron energies,  $E_{e1} \simeq$  several tens of MeV, and their relativistic electrons rotate many times along orbits that are close to circular. Such storage-ring SRs are smaller, their sizes are of the order of several metres, they are cheaper and can generate larger beam currents. TR emitted from a low-energy storage-ring SR could be used for performing X-ray lithography (XRL) (Piestrup, Powell *et al.*, 1998; Yamada, 1990).

The necessity of many SR customers to have their own small SR with large electron beam current has resulted in a pursuit for development of even smaller storage-ring SRs, with  $E_{e1} \simeq 10$  MeV. Our group utilizes and produces the table-top

storage-ring SR MIRRORCLE-20SX, with a relativistic electron energy of  $E_{el} = 20$  MeV (<http://www.ppl-xray.com>). MIRRORCLE-20SX is the most powerful table-top SR in the world that can be used for XRL.

The cut-off energy of the emitted TR, above which it quickly decreases with decreasing  $E_{el}$ , and of LINACs with  $E_{el} = 1$  GeV, is  $E_{co} \simeq 59$  keV, while, for storage-ring SRs with  $E_{el} = 20$  MeV, it is  $E_{co} \simeq 1.2$  keV. For all materials used for preparation of TR targets there are no absorption edges for photon energies above 10 keV, but there are many absorption edges around and below 1.2 keV (National Institute of Standards and Technology, <http://physics.nist.gov/PhysRefData/FFast/html/form.html>). Therefore, the dielectric constant of the target material should be considered to be a complex number (Attwood, 1999) for low-energy storage-ring SRs, although in the existing theory of TR the dielectric constant is a real number. Also, there is no formula describing the number of passes  $N_p$  through a TR target of a relativistic electron injected in the storage ring, and therefore the beam current is unknown analytically. Furthermore, there is no theoretical approach for quantification of the XRL efficiency of a TR target. Correspondingly, optimization of TR targets for storage-ring SRs has not been carried out.

In this paper a theory of emission of TR from low-energy electron storage-ring SRs is proposed. It takes into account the fact that the dielectric constant of the TR target material is a complex number, proposes an analytical expression for  $N_p$  and an approach for determining the XRL efficiency of a TR target. This theory can be used for designing TR targets for XRL.

According to the existing theory, for emission of TR by LINACs the following results are valid for one relativistic electron which is incident perpendicular to a foil TR target, and passes once through it:

(i) The boundary conditions of the wavevector of the emitted TR at the interface between foil and vacuum are (Durand, 1975)

$$\begin{aligned} \mathbf{k}_1 &= \varepsilon_1^{1/2} \frac{\omega}{c} \hat{\mathbf{k}}_1, & k_{1z} &= \frac{\omega}{c} (\varepsilon_1 - \sin^2 \theta)^{1/2}, \\ \mathbf{k}_{1\perp} &= \frac{\omega}{c} \sin \theta \times \hat{\mathbf{k}}_{1\perp}, & \mathbf{k}_2 &= \frac{\omega}{c} \hat{\mathbf{k}}_2, \\ k_{2z} &= \frac{\omega}{c} \cos \theta, & \mathbf{k}_{2\perp} &= \frac{\omega}{c} \sin \theta \times \hat{\mathbf{k}}_{2\perp}, \end{aligned} \quad (1)$$

where the electron moves along the  $z$ -axis which is perpendicular to the foil surface,  $\mathbf{k}_{1\perp}$  and  $k_{1z}$  are the components of the wavevector  $\mathbf{k}_1$  at the foil surface along that surface and the  $z$ -axis, respectively,  $\mathbf{k}_{2\perp}$  and  $k_{2z}$  are the components of the wavevector  $\mathbf{k}_2$  at the vacuum surface along that surface and the  $z$ -axis, respectively,  $\varepsilon_1$  is the dielectric constant of the foil, the energy of an emitted TR photon of frequency  $\omega$  is  $E$  (eV) =  $\hbar\omega(1/s)$  where  $\hbar = h/2\pi = 6.5825 \times 10^{-16}$  eV s, and  $\theta$  is the emission angle with respect to the direction of the incident electron.

(ii) The dielectric constant of the foil has been approximated as (Durand, 1975; Piestrup *et al.*, 1985)

$$\varepsilon_1(E) = 1 - (E_1/E)^2, \quad (2)$$

where  $E_1$  is the plasma energy of the foil. This approximation is valid when  $E \gg E_{ion\max}$ , where  $E_{ion\max}$  is the maximum electron ionization energy for the chemical elements of the target.

(iii) The pass of one relativistic electron through a foil induces electronic polarization of the foil material, and emission of TR which can be expressed using its vector potential. The vector potential of the emitted TR by one foil in a vacuum (Durand, 1975),

$$\begin{aligned} \mathbf{A}(\mathbf{r}, \omega) &= \left[ \frac{i\mu_0}{4\sqrt{2}\pi^{3/2}} \frac{e\omega}{v} \frac{\mathbf{k}_{1\perp}}{c k_{1\perp}^2 + (\omega/\gamma v)^2} \frac{\exp(ik_2 r)}{r} \frac{\varepsilon_1 - 1}{\omega/v - k_{1z}} \right] \\ &\times \{1 - \exp[i(k_{1z} - \omega/v)d]\} \\ &= \mathbf{A}_s \times C_i, \end{aligned} \quad (3)$$

describes the TR at a point determined by the vector  $\mathbf{r}$  starting at the emission point, where  $d$  is the foil thickness,  $e$  and  $v$  are the charge and the speed of the relativistic electron, respectively,  $\gamma = (1 - v^2/c^2)^{-1/2} = E_{el}/E_{el\text{rest}}$ , where  $E_{el\text{rest}}$  is the rest electron energy,  $\mathbf{A}_s$  is the vector potential due to the emission from one foil/vacuum interface, and  $C_i$  formulates the TR interference between the two emitting interfaces of the foil.

(iv) Since a target for one electron pass should contain a large number of foils  $M$ , it is considered that TR photons, emitted around the foil's first emitting interface, are not absorbed while these photons pass through that foil (Durand, 1975; Piestrup *et al.*, 1985).

(v) The differential photon efficiency for emission from one foil (Piestrup *et al.*, 1985; Garibyan *et al.*, 1974),

$$\begin{aligned} \frac{d^2 N_f}{d\Omega dE} \left( \frac{1}{\text{eV}} \right) &= \frac{|\mathbf{S}_p| r^2}{\hbar} = \frac{c}{2\mu_0} \frac{|\mathbf{k}_2 \times \mathbf{A}|^2 r^2}{\hbar} \\ &= \left[ \frac{c}{2\mu_0} \frac{|\mathbf{k}_2 \times \mathbf{A}_s|^2 r^2}{\hbar} \right] |C_i|^2 = F_s F_i \\ &= \left[ \frac{\alpha \sin^2 \theta}{\pi^2 E l^2} (L_1 - L_2)^2 \right] [4 \sin^2(d/L_1)], \end{aligned} \quad (4)$$

represents the probability of emitting a photon, within the energy range  $[\hbar(\omega - d\omega/2, \omega + d\omega/2)]$ , and the spatial angle  $d\Omega = 2\pi \sin\theta d\theta$ , as a result of one pass of one relativistic electron through one foil, and  $l = 4\hbar c/E$ . This probability is determined by the Poynting vector  $\mathbf{S}_p$ , and it contains a multiplier  $F_s$  giving the contribution of one foil/vacuum interface, and a multiplier  $F_i$  formulating the TR interference between the two emitting interfaces of the foil. The formation lengths  $L_1$  and  $L_2$  which depend on the plasma energies  $E_1$  and  $E_2$  are (Piestrup *et al.*, 1985)

$$\begin{aligned} L_i(E, \theta) &\simeq \frac{l}{1/\gamma^2 + \theta^2 + (E_i/E)^2}, \\ E_i &= \left( \frac{\hbar^2 e^2 N_A}{\varepsilon_0 m} \right)^{1/2} \left( \frac{n_i \rho_i}{A_i} \right)^{1/2} \text{ for } i = 1, 2, \end{aligned} \quad (5)$$

where  $m$  is the rest mass of the electron,  $A_i$  is the atomic weight,  $n_i \cong Z_i$  is the number of free electrons per atom  $\cong$



$$\begin{aligned}
 \frac{d^2 N_f}{d\Omega dE} &= \frac{c}{2\mu_0} \frac{|\mathbf{k}_2 \times \mathbf{A}|^2 r^2}{\hbar} = \left[ \frac{c}{2\mu_0} \frac{|\mathbf{k}_2 \times \mathbf{A}_s|^2 r^2}{\hbar} \right] |C_i|^2 |D_2|^2 \\
 &= F_s F_i F_D \\
 &= \left[ \frac{\alpha \sin^2 \theta}{\pi^2 E l^2} |\dot{L}_1 - \dot{L}_2|^2 \right] \left\{ [1 - \exp(-4\beta_1 d/l)]^2 \right. \\
 &\quad \left. + 4 \exp(-4\beta_1 d/l) \sin^2 [\text{Re}(d/\dot{L}_1)] \right\} [\exp(-8\beta_2 r/l)], \quad (10)
 \end{aligned}$$

where the formation lengths are complex,

$$\dot{L}_i(E, \theta) \cong \frac{l}{1/\gamma^2 + \theta^2 + 2(\delta_i - i\beta_i)}$$

and

$$\begin{aligned}
 |\dot{L}_1 - \dot{L}_2|^2 &= \frac{4l^2 [(\delta_1 - \delta_2)^2 + (\beta_1 - \beta_2)^2]}{[(1/\gamma^2 + \theta^2 + 2\delta_1)^2 + (2\beta_1)^2][(1/\gamma^2 + \theta^2 + 2\delta_2)^2 + (2\beta_2)^2]} \\
 &\text{for } i = 1, 2. \quad (11)
 \end{aligned}$$

In the case of one foil in a vacuum,  $\delta_2 = 0$ ,  $\beta_2 = 0$  and

$$\begin{aligned}
 \frac{d^2 N_f}{d\Omega dE} &= F_s F_i = \left[ \frac{\alpha \sin^2 \theta}{\pi^2 E l^2} |\dot{L}_1 - L_2|^2 \right] \\
 &\times \left\{ \left[ 1 - \exp\left(-4\beta_1 \frac{d}{l}\right) \right]^2 + 4 \exp\left(-4\beta_1 \frac{d}{l}\right) \sin^2\left(\text{Re} \frac{d}{\dot{L}_1}\right) \right\} \\
 &= \left\{ \frac{4\alpha \sin^2 \theta}{\pi^2 E} \frac{(\delta_1^2 + \beta_1^2)}{[(1/\gamma^2 + \theta^2 + 2\delta_1)^2 + (2\beta_1)^2](1/\gamma^2 + \theta^2)^2} \right\} \\
 &\times \left\{ [1 - \exp(-4\beta_1 d/l)]^2 \right. \\
 &\quad \left. + 4 \exp(-4\beta_1 d/l) \sin^2 \left[ \frac{d}{l} (1/\gamma^2 + \theta^2 + 2\delta_1) \right] \right\}. \quad (12)
 \end{aligned}$$

### 2.3. Emission of TR by a low-energy storage-ring SR

When a TR target is located in a storage-ring SR, experimental studies (Piestrup, Lombardo *et al.*, 1998; Kaplin *et al.*, 2002) show that an injected electron passes more than once through the target before its dissipation. There is no analytical expression though for the number of passes  $N_p$  of an injected electron through a TR target. There are three main mechanisms of energy loss in the target which determine this dissipation, and correspondingly  $N_p$ , of an injected electron, namely bremsstrahlung, TR and electrostatic scattering from the nucleus of the atoms of the target material (Yamada, 1996).

The contribution of bremsstrahlung, however, decreases when decreasing the thickness of a one-foil TR target, and for submicrometre foils the ratio of the emitted bremsstrahlung photon energy to the emitted TR energy does not exceed 0.5% (Piestrup, Boyers, Pincus, Harris, Maruyama *et al.*, 1991; Piestrup, Boyers, Pincus, Harris, Caplan *et al.*, 1991). Furthermore, the emitted TR energy does not exceed 0.1% of the injected electron energy (Piestrup, Boyers, Pincus, Harris,

Maruyama *et al.*, 1991; Piestrup, Boyers, Pincus, Harris, Caplan *et al.*, 1991). Therefore, the number of passes  $N_p$  of an injected electron is determined by its electrostatic scattering from the target.

According to the experimental study (Piestrup, Lombardo *et al.*, 1998) of TR from a one-foil target,  $N_p \cong 1/d$ . Furthermore, the cross section of electrostatic scattering is proportional to  $\rho_1 Z_1^2 / (A_1 E_{el}^2)$  (Yamada, 2003; Yavosky & Detlaf, 1977). Correspondingly, the number of passes  $N_p$  of an injected electron through a TR target of  $M$  identical foils can be represented as

$$N_p = C(A_1 E_{el}^2) / (\rho_1 Z_1^2 M d), \quad (13)$$

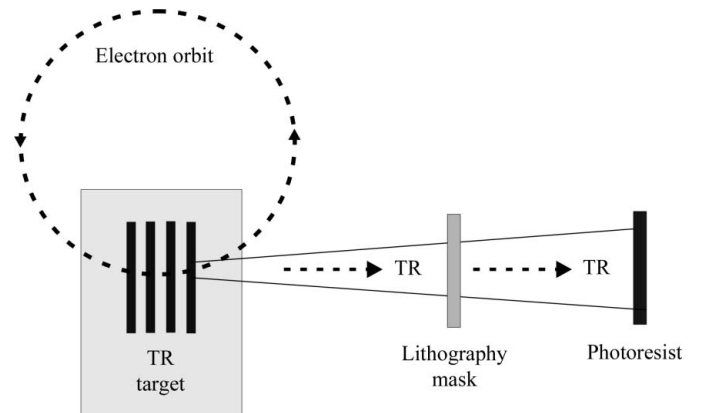
where  $C$  is a constant for a given storage-ring SR. Indeed, analysis of the experimental results about  $N_p$  (Piestrup, Lombardo *et al.*, 1998; Kaplin *et al.*, 2002) obtained for a one-foil target made of different chemical elements, in several storage-ring SRs, shows that those  $N_p$  are described well by (13).

Consequently, the differential photon efficiency of a TR target, consisting of  $M$  identical foils, inserted in a low-energy storage-ring SR is

$$\begin{aligned}
 \frac{d^2 N}{d\Omega dE} &= F_s F_i F_M N_p \\
 &= \left\{ \frac{4\alpha \sin^2 \theta}{\pi^2 E} \frac{(\delta_1^2 + \beta_1^2)}{[(1/\gamma^2 + \theta^2 + 2\delta_1)^2 + (2\beta_1)^2](1/\gamma^2 + \theta^2)^2} \right\} \\
 &\times \left\{ (1 - \exp[-(E/\hbar c)\beta_1 d])^2 + 4 \exp[-(E/\hbar c)\beta_1 d] \right. \\
 &\times \sin^2 \left[ \frac{Ed}{4\hbar c} (1/\gamma^2 + \theta^2 + 2\delta_1) \right] \left. \right\} \left[ \frac{1 - \exp(-M\sigma)}{1 - \exp(-\sigma)} \right] \\
 &\times \left( \frac{CA_1 E_{el}^2}{\rho_1 Z_1^2 M d} \right). \quad (14)
 \end{aligned}$$

### 2.4. Sensitivity of XRL photoresist to TR emitted by a storage-ring SR

A simplified illustration of performing XRL by a storage-ring SR is shown in Fig. 2. The emitted TR passes through a



**Figure 2**  
Simplified illustration of performing XRL by a storage-ring SR.

lithography mask and is partially absorbed in the photoresist. The resist sensitivity  $S_R$  for one injected electron passing through a multi-foil TR target is approximated as

$$S_R = N_p \iint [F_s(E, \Omega) F_i(E, \Omega) d\Omega] F_M(E) F_c(E) dE, \quad (15)$$

where  $F_c(E) = \exp(-\mu_{\text{mask}} d_{\text{mask}}) \mu_{\text{resist}}$ ,  $\mu_{\text{mask}}(E)$  is the linear absorption coefficient of the mask,  $d_{\text{mask}}$  is the mask thickness, and  $\mu_{\text{resist}}(E)$  is the linear absorption coefficient of the resist.  $F_c$  is a function representing the transmission of TR through the lithography mask, and its partial absorption in the photoresist. Its formulation is in accordance with the experimental result (Seligson *et al.*, 1988) that the sensitivity of some XRL photoresists depends uniquely on the absorbed dose density ( $\text{mJ cm}^{-3}$ ) of TR, independent of the photon energy. Since  $S_R$  should be as large as possible, designing TR targets for XRL can be based on maximization of  $S_R$ . In such a design, the thicknesses and the number of layers of the TR target are chosen to provide a maximum of  $S_R$  from (15).

### 3. Calculated results and their analysis

To determine the characteristics of TR emitted by a low-energy SR, calculations have been performed to emulate TR targets in our storage-ring SR MIRRORCLE-20SX, with  $\gamma = E_{\text{el}}/E_{\text{el,rest}} = 20 \text{ (MeV)}/0.511 \text{ (MeV)} \cong 39.139$ . Taking into account the known characteristics of the angular distribution of TR emission, as well as the fact that it has a maximum emission at  $\theta \cong \theta_{\text{max}} = 1/\gamma \cong 25.55 \text{ mrad}$ , the integrations over the spatial angle are performed for  $\theta = [0, 500] \text{ mrad}$ . The presented results refer to only one relativistic electron.

The spectral dependence of the emission of TR by one material/vacuum interface is characterized by the function

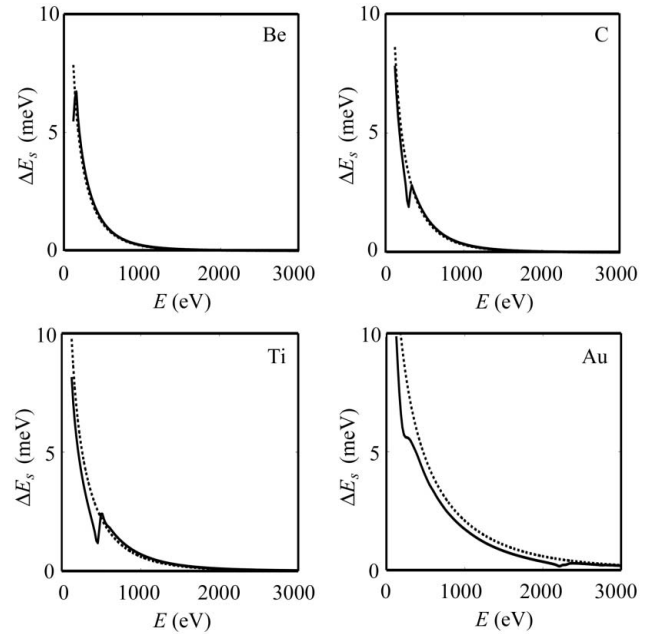
$$\begin{aligned} \Delta E_s(E) &= E \int_{E-0.5 \text{ eV}}^{E+0.5 \text{ eV}} \left( \int \frac{d^2 N_s}{d\Omega dE} d\Omega \right) dE \\ &= E \left[ \int F_s(\Omega, E) d\Omega \right] \times (1 \text{ eV}), \end{aligned} \quad (16)$$

which represents the energy of the emitted TR photons, with energy within 1 eV around the photon energy  $E = \hbar\omega$ , for one pass of one relativistic electron through that interface. The dependence  $\Delta E_s(E)$  is shown in Fig. 3 for the materials Be, C, Ti and Au. The results obtained under the simplified assumption  $\varepsilon_1(E) = 1 - (E_1/E)^2$  are calculated from (4) and (5), and are shown by dotted lines. The corresponding results obtained under the precise assumption  $\varepsilon_1(E) = 1 - 2[\delta_1(E) - i\beta_1(E)]$  are calculated from (12) and are shown by solid lines.

Be is known to have electron ionization energy at  $E_{\text{ion}}(\text{Be}) = 111 \text{ eV}$ , while  $E_{\text{ion}}(\text{C}) = 248 \text{ eV}$ ,  $E_{\text{ion}}(\text{Ti}) = 564 \text{ eV}$  and  $E_{\text{ion}}(\text{Au}) = [334, 762] \text{ eV}$  as well as  $E_{\text{ion}}(\text{Au}) = [2005, 2291] \text{ eV}$ . Fig. 3 shows that the precise assumption that  $\varepsilon_1(E)$  is complex leads to a notable change of the TR emission spectrum around the ionization energies of the atom. The behaviour of the above TR emission spectra can be explained by an analysis of the known spectral dependencies of the real and imaginary part of the atomic scattering factor  $f^0(E) = f_1^0(E) - if_2^0(E)$ ,

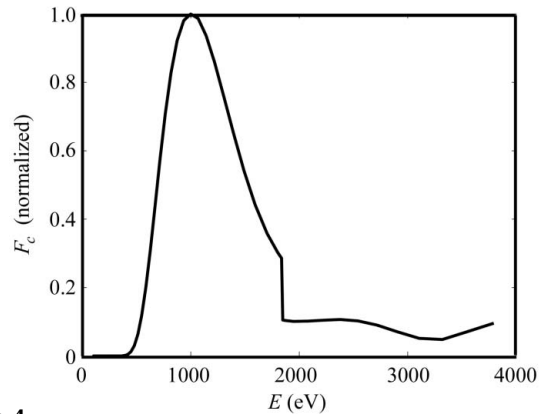
according to (8). Therefore, the increase of  $\Delta E_s(E)$  just above  $E_{\text{ion}}$ , compared with just below  $E_{\text{ion}}$ , is due to stronger absorption of TR emission just above  $E_{\text{ion}}$ , which leads to additional ionization of the atoms, larger electronic polarization of the material and larger TR emission.

The function  $F_c(E) = \exp(-\mu_{\text{mask}} d_{\text{mask}}) \mu_{\text{resist}}$  represents the transmission of TR through the XRL lithography mask, and its partial absorption in the XRL photoresist. The function  $F_c(E)$ , normalized to unity, is presented in Fig. 4 for a SiC mask of thickness 2  $\mu\text{m}$ , and PMMA photoresist, which are typically used for XRL. It is seen that TR emitted within the photon energy range  $E = [490, 1860] \text{ eV}$  is most useful for performing XRL with such a mask and resist. TR photons with energy



**Figure 3**

Spectral dependence of the emitted TR energy, (16), from one interface between material (Be, C, Ti and Au) and vacuum, for one pass of one relativistic electron through that interface. The results calculated assuming that the dielectric constant is a real function are shown by dotted lines, and those calculated assuming that the dielectric constant is a complex function are shown by solid lines.



**Figure 4**

Dependence of the function  $F_c$  on the energy  $E$  of the emitted TR photons.  $F_c(E)$  is normalized to unity for a SiC mask of thickness 2  $\mu\text{m}$  and PMMA photoresist

**Table 1**

Emitted TR energy  $E_s(E_b, E_c) = \int_{E_b}^{E_c} E [\int F_s(\Omega, E) d\Omega] dE$  from the interface between several pairs of materials, for one pass of one relativistic electron through that interface.

Interface	Al/Si	Al/Au	Be/Si	Be/Au	Al/vacuum	Be/vacuum	Au/vacuum
$E_s$ (eV) for $[E_b, E_c] = [490, 1860]$ eV	0.00287	0.700	0.0103	0.903	0.544	0.382	2.228
$E_s$ (eV) for $[E_b, E_c] = [80, 4000]$ eV	0.1121	1.117	0.1846	1.478	2.277	1.724	4.888

$E < 490$  eV are absorbed in the mask, while the vast majority of photons with energy  $E > 1860$  eV are not absorbed in the resist but pass through it. It is also seen that TR emission with  $E \cong 1$  keV is most suitable for performing such XRL.

The presence of the function  $F_c(E)$  as a multiplier in (15) nullifies the contribution of the emitted TR with  $E < 490$  eV to the resist sensitivity  $S_R$ , although the spectral dependence of the TR emission increases strongly for smaller  $E$  (see Fig. 3). Correspondingly, TR targets can be optimized, for the first time, and this can be done by maximization of  $S_R$ .

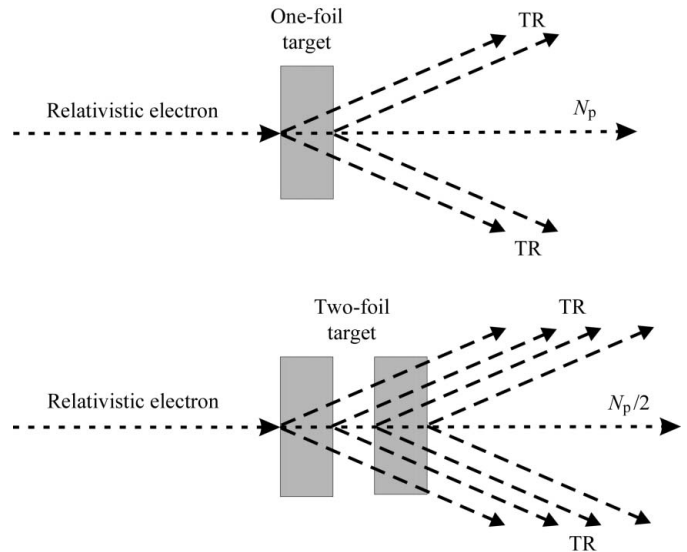
Taking into account the characteristics of both the function  $F_c(E)$  and the TR emission spectra from Fig. 3, most of the following integrations over the photon energy are performed within the energy interval  $[E_b, E_c] = [80, 4000]$  eV. To compare the TR energy emitted from different interfaces, the following function is used,

$$E_s(E_b, E_c) = \int_{E_b}^{E_c} E \left[ \int F_s(\Omega, E) d\Omega \right] dE,$$

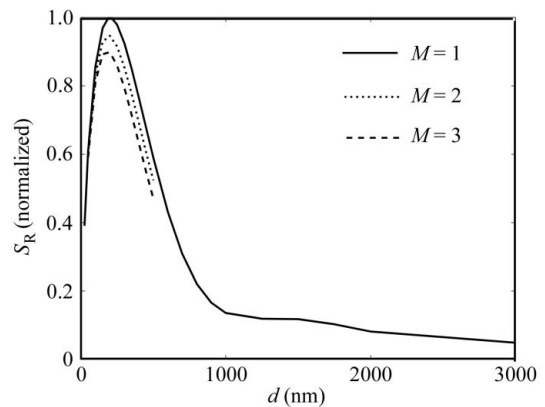
and its values are calculated from (10) and (11). Results for  $E_s(E_b, E_c)$  are included in Table 1 for several different interfaces.

These results confirm that the material/vacuum interface emits more TR than the material/(lighter material) interface (Chu *et al.*, 1980). Indeed, analysis of (10) and (11) shows that this is so because the numerator of the expression for  $|\dot{L}_1 - \dot{L}_2|$  has a larger value for the material/vacuum interface. The reason for the above is that TR is generated as a result of the difference between the electronic polarizations of the materials on the two sides of the interface, and this difference is larger for the material/vacuum interface. Correspondingly, TR targets should be made of foils rather than of solid-state multilayer targets. Therefore, from now on we will discuss only multi-foil TR targets.

A schematic drawing of TR emission from a one-foil target and two-foil target in a storage-ring SR is illustrated in Fig. 5. These foils are assumed to be identical. In the case of a two-foil target, TR is emitted from twice as many foil/vacuum interfaces (four foil/vacuum interfaces) compared with a one-foil target. For the two-foil target though, one injected electron passes half as many times through the target compared with a one-foil target, according to (13). Consequently, the product  $MN_p$  is the same for a two-foil target and one-foil target. For the two-foil target, however, TR emitted from the first foil (on the left in Fig. 5) passes through the second foil and is partially absorbed there. Therefore, the one-foil target should emit more TR energy than the two-foil target. Corre-



**Figure 5** Schematic illustration of TR emission from a one-foil target and two-foil target as a result of the injection of one relativistic electron into a storage-ring SR. These foils are assumed to be identical.



**Figure 6** Dependence of the XRL resist sensitivity  $S_R$  as a function of the foil thickness for identical Al foil TR target in MIRRORCLE-20SX. XRL is assumed to be performed using a SiC mask of thickness  $2 \mu\text{m}$ , and PMMA resist.  $S_R$  is normalized to unity for the one-foil target, and results are presented for targets containing one, two and three foils.

spondingly, the fewer foils the TR target contains, the more TR energy it should emit.

To determine the optimum foil thickness for a foil target, the dependence of the resist sensitivity  $S_R$  (15) on the foil thickness  $d$  is shown in Fig. 6 for an Al foil target in MIRRORCLE-20SX. XRL is assumed to be performed using a SiC mask of thickness  $2 \mu\text{m}$ , and PMMA resist.  $S_R$  is normalized to unity for the one-foil target, and the results are presented for  $M = 1, 2$  and  $3$ .

It is seen that the dependence  $S_R(d)$  has a relatively sharp maximum at  $d \cong 200$  nm. Furthermore, it is confirmed that a one-foil target emits most TR energy, while a two-foil target emits more energy than a three-foil target. Therefore, the best Al-based TR target for MIRRORCLE-20SX is a one-foil Al target with an optimum foil thickness of  $d_{\text{opt}} \cong 200$  nm.

In the existing theory of TR emission, developed for one pass of the injected relativistic electrons through identical foil targets in high-energy SR (Piestrup *et al.*, 1985; Piestrup, Boyers, Pincus, Harris, Maruyama *et al.*, 1991), the optimum foil thickness has been determined from the condition  $d_{\text{opt}_{\text{sin}}} = \pi L_1(E_{\text{max}}, \theta_{\text{max}})/2$ , which ensures a maximum of the function  $F_i = 4 \sin^2[d/L_1(E_{\text{max}}, \theta_{\text{max}})]$  in (6). For MIRRORCLE-20SX,  $L_1(E_{\text{max}}, \theta_{\text{max}}) \cong L_1(1 \text{ keV}, 25.5 \text{ mrad}) \cong 331.6$  nm, and the above condition gives  $d_{\text{opt}_{\text{sin}}} = \pi L_1(1 \text{ keV}, 25.5 \text{ mrad})/2 \cong 520.9$  nm.

Apparently, for a target with a fixed number of foils  $M$ ,  $d_{\text{opt}}$ , which we obtain by maximization of the dependence  $S_R(d)$  using (14) and (15), is smaller than the formation length  $L_1(E_{\text{max}}, \theta_{\text{max}})$  for the foil, and much smaller than  $d_{\text{opt}_{\text{sin}}}$  determined by maximization of the function  $F_i = 4 \sin^2[d/L_1(E_{\text{max}}, \theta_{\text{max}})]$  in (6). Correspondingly,  $d_{\text{opt}} < L_1(E_{\text{max}}, \theta_{\text{max}}) < d_{\text{opt}_{\text{sin}}}$ . The relation  $d_{\text{opt}} \ll d_{\text{opt}_{\text{sin}}}$  is due to the fact that, in the presented theory, the number of passes  $N_p$  of the injected electron increases for thinner foils, according to (13). Utilization of foils with  $d_{\text{opt}} < L_1(E_{\text{max}}, \theta_{\text{max}})$  does not contradict the theory of TR emission because, according to Durand (1975), there is still some TR emission when the above condition is satisfied. In this case, however, the emitted TR energy from a foil is relatively small, and  $E \simeq d^2$ .

#### 4. Discussion

In this paper a theory of emission of TR by low-energy electron storage-ring SRs is proposed. It takes into account the fact that the dielectric constant of the target material is a complex number as a result of the low energy of the emitted TR. An analytical expression is presented for the number of passes  $N_p$  through a TR target of a relativistic electron injected in the storage ring. This represents a basis for analytical determination of the beam current, and the emitted TR power from the target.

TR emitted by a low-energy storage-ring SR can be used for performing XRL. Therefore, an analytical expression is introduced for the photoresist sensitivity  $S_R$  obtained for one injected electron passing through a multi-foil TR target. Maximization of  $S_R$  with respect to the number of foils of the

target, their thicknesses and material could be used for finding out an optimum design of the TR target for XRL.

Analysis of the calculated results indicates that, for a particular target material, most TR power is emitted from a target containing only one foil of that material. It is shown that for an Al target, in a storage-ring SR with  $E_{\text{el}} = 20$  MeV, a target containing only one Al foil of thickness about 200 nm would be best for performing XRL.

#### References

- Attwood, D. (1999). *Soft X-rays and Extreme Ultraviolet Radiation*, pp. 52–61. Cambridge University Press.
- Cherry, M. L., Hartmann, G., Muller, D. & Prince, T. A. (1974). *Phys. Rev. D*, **10**, 3594–3607.
- Chu, A. N., Piestrup, M. A., Barbee, T. W. Jr & Pantell, R. H. (1980). *J. Appl. Phys.* **51**, 1290–1293.
- Durand, L. (1975). *Phys. Rev. D*, **11**, 89–105.
- Ebert, P. J., Moran, M. J., Dahling, B. A., Berman, B. L., Piestrup, M. A., Kephart, J. O., Park, H., Klein, R. K. & Pantell, R. H. (1985). *Phys. Rev. Lett.* **54**, 893–896.
- Garibyan, G. M. (1961). *Sov. Phys. JETP*, **12**, 237–239.
- Garibyan, G. M., Gevorgyan, L. A. & Yang, C. (1974). *Sov. Phys. JETP*, **39**, 265–270.
- Jackson, J. D. (1999). *Classical Electrodynamics*. New York: John Wiley.
- Kaplin, V. V., Uglov, S. R., Bulaev, O. F., Goncharov, V. J., Voronin, A. A., Piestrup, M. A. & Gary, C. K. (2002). *Rev. Sci. Instrum.* **73**, 63–68.
- Pianetta, P., Redaelli, R., Jaeger, R. & Barbee, T. W. (1985). *Proc. SPIE*, **537**, 69–73.
- Piestrup, M. A., Boyers, D. G., Pincus, C. I., Harris, J. L., Caplan, H. S., Silzer, R. M. & Skopik, D. M. (1991). *Appl. Phys. Lett.* **59**, 189–191.
- Piestrup, M. A., Boyers, D. G., Pincus, C. I., Harris, J. L., Maruyama, X. K., Bergstrom, J. C., Caplan, H. S., Silzer, R. M. & Skopik, D. M. (1991). *Phys. Rev. A*, **43**, 3653–3661.
- Piestrup, M. A., Kephart, J. O., Park, H., Klein, R. K., Pantell, R. H., Ebert, P. J., Moran, M. J., Dahling, B. A. & Berman, B. L. (1985). *Phys. Rev. A*, **32**, 917–927.
- Piestrup, M. A., Lombardo, L. W., Cremer, J. T., Retzlaff, G. A., Silzer, R. M., Skopik, D. M. & Kaplin, V. V. (1998). *Rev. Sci. Instrum.* **69**, 2223–2229.
- Piestrup, M. A., Powell, M. W., Cremer, J. T., Lombardo, L. W., Kaplin, V. V., Mihal'chuk, A. A., Uglov, S. R., Zabaev, V. N., Skopik, D. M., Silzer, R. M. & Retzlaff, G. A. (1998). *Proc. SPIE*, **3331**, 450–463.
- Seligson, D., Pan, L., King, P. & Pianetta, P. (1988). *Nucl. Instrum. Methods*, **A266**, 612–618.
- Ter-Mikaelyan, M. L. (1961). *Nucl. Phys.* **24**, 43–61.
- Yamada, H. (1990). *J. Vac. Sci. Technol.* **B8**, 1628–1631.
- Yamada, H. (1996). *Jpn. J. Appl. Phys.* **35**, L182–L185.
- Yamada, H. (2003). *Nucl. Instrum. Methods*, **B199**, 509–516.
- Yavorsky, B. M. & Detlaf, A. A. (1977). *Handbook of Physics*. Moscow: Science.